Asymmetric information, liquidity constraints, and efficient trade

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Allocation problem

Consider the problem of efficiently (re)allocating a good/asset among n agents (e.g. public project, natural resource).

Usual challenges: Reveal information and ensure voluntary participation.

Overlooked challenge: Ensure feasibility of payments.

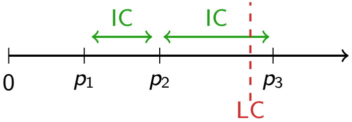
- Agents may be financially constrained.

Incentive compatibility and liquidity constraints

Revealing private information is usually done by letting agents freely pick an option in a price schedule.

We want that agents with **different valuations choose different options.**

Liquidity constraints require that prices stay *low enough*.



Centralized markets

When designing **centralized markets,** there is some leeway in dealing with the liquidity problem.

We can **subsidize the most liquidity-constrained agents** with the resources of the least liquidity-constrained ones.

- Lump-sum transfers.
- Possible until it conflicts with voluntary participation.

In general, only a partial solution and **fails to address the most constrained cases.**

- The design of the IC constraints is crucial here.
- A *better* design reduces the need for lump-sum transfers.

The environment

A set $N := \{1, \ldots, n\}$ of agents, **privately informed** about $v_i \in V_i$.

Agents have **liquidity resources** $l := (l_1, \ldots, l_n) \in \mathbb{R}^n_+$.

Each agent has **outside option**: $u_i^0 : V \to \mathbb{R}$, where $V = \times_{i \in N} V_i$.

Ex post net utility of agent i:

$$v_i s_i + t_i - u_i^0.$$

(re)Allocation mechanisms

A **mechanism** is a pair $(s, t) := (s_1, \ldots, s_n, t_1, \ldots, t_n)$ where,

$$\text{Allocation rule:} \quad s_i : V \to [0, 1],$$
$$\text{Transfer rule:} \quad t_i : V \to \mathbb{R}.$$

A mechanism is (ex post) **efficient** when $s_i(v) = \mathbb{1}\{v_i = \max_j v_j\}$.

Simple questions:

1. Under which conditions an efficient mechanism exists?
2. What do they look?
3. How features of the environment such as market size or initial ownership affect them?

An (oversimplified) example

n agents, v_i's iid $\mathcal{U}[0,1]$.

Assume that we can choose between:

- second-price auction;
- first-price auction.

In both cases

1. Full revelation of information;
2. The allocation is efficient;
3. Same interim expected payments (revenue equivalence theorem).

Example: Bidding strategies

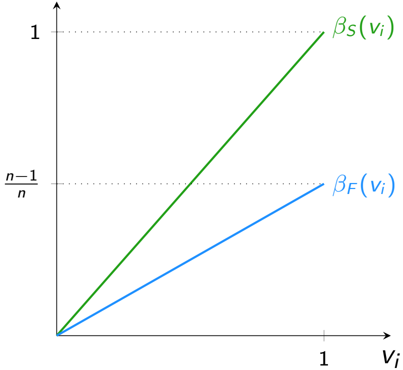
What differs between the two auctions are the **bidding strategies.**

At equilibrium:

$\beta_S(v_i) = v_i$
$\beta_F(v_i) = \frac{n-1}{n} v_i$

Clearly,

$\beta_S(v_i) \in [0, 1]$
$\beta_F(v_i) \in [0, \frac{n-1}{n}]$



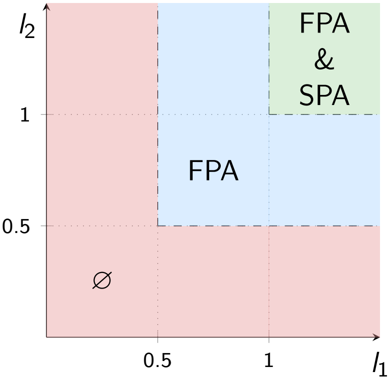
Example: Feasible payments without redistribution

When can we be sure that any type $v_i \in [0,1]$ can submit a bid **they can actually afford?**

Assume $n = 2$.
$l_i \in \mathbb{R}_+$: agent i's liquidity.

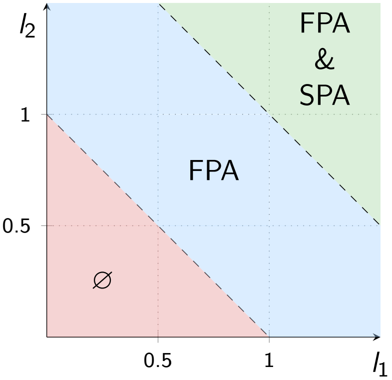
We have,

$\beta_S(v_i) \in [0,1]$
$\beta_F(v_i) \in [0, \frac{1}{2}]$



Example: Feasible payments with redistribution

Assume that a designer could enforce **any ex ante redistribution** of liquidity between agents.



Example: Takeaway #1

Different (efficient) incentive compatible mechanisms **require different levels of individual liquidity.**

- SPA is more demanding than FPA.

Ex ante redistribution of liquidity **alleviates the problem.**

- More *effective* in FPA than in SPA.
- Lower aggregate liquidity needed in FPA.
- Hence the choice of IC mechanism matters.

Example: Market size

The **number of participants** can also affect the liquidity requirement of a mechanism.

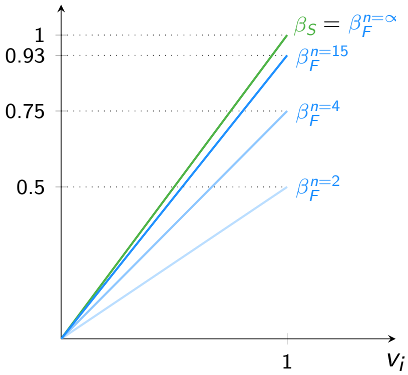
Recall that when $n \geq 2$:

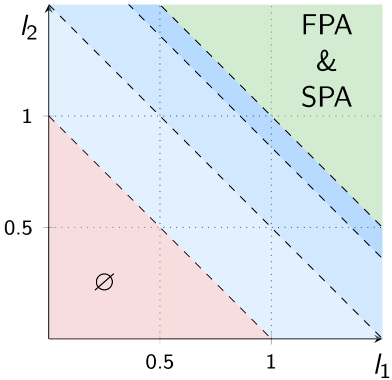
- $\beta_F(v_i) = \frac{n-1}{n} v_i$: **increasing in n.**
- $\beta_S(v_i) = v_i$

Hence, more bidders in FPA $\Rightarrow$ increases individual liquidity requirements.

Large market: When $n = \infty$, $\beta_F = \beta_S$.

Example: Market size





Example: Takeaway #2

More participants make **liquidity requirements stronger.**

When $n = \infty$, both auctions impose $l_i \geq 1$ for $i = 1, 2$.

- 1 is is the **largest possible valuation** as $v_i \in [0, 1]$.

What's missing, what's next?

What is **omitted** in the example? Pretty much everything.

- Focus on two particular mechanisms;
- Unrestricted redistribution of liquidity;
- No participation constraints/outside option;
- Unbalanced transfers.

But all the previous **intuitions hold in the general trading environment.**

Related literature

Optimal auctions with liquidity constraints: Laffont and Robert (1996), Malakhov and Vohra (2008), Boulatov and Severinov (2021).

FPA with private budgets: Kotowski (2020), Bobkova (2020).

Allocation problems with two-sided private information: Myerson and Satterthwaite (1983), Cramton et al. (1987), Loertscher et al. (2015), Loertscher and Wasser (2019).

Back to the general environment

We want to impose four constraints on our mechanisms:

1. Incentive compatibility: $U_i(v_i) \geq U_i(\hat{v}_i; v_i)$
2. Liquidity constraints: $t_i(v) \geq -l_i$
3. Participation constraints: $U_i(v_i) \geq 0$
4. Budget balance: $\sum_i t_i(v) = 0$.

IC and participation are interim.

Liquidity and BB are ex post.

Expected externality mechanism

How to set transfers to satisfy IC, participation and BB?

Usual candidate: **Expected externality mechanism**.

$$\tilde{t}_i(v) := \tilde{\varphi}_i(v) - \frac{1}{n-1} \sum_{j \neq i} \tilde{\varphi}_j(v) + \tilde{\phi}_i,$$

where $\tilde{\varphi}_i(v) := \mathbb{E}_{-i} \sum_{j \neq i} v_j s_j^*(v)$ and $\sum_{i \in N} \tilde{\phi}_i = 0$.

This transfer rule has range $2\mathbb{E}[\max_{j \neq i} v_j]$.

The main result shows that we can do better than this.

Expected externality mechanism: modified

Consider the following **modified** expected externality mechanism:

$$t_i(v) := \varphi_i(v) - \frac{1}{n-1} \sum_{j \neq i} \varphi_j(v) + \phi_i,$$

where $\sum_{i \in N} \phi_i = 0$, and

$$\varphi_i(v) := \frac{n-1}{n} \sum_{j \neq i} \mathbb{E}[\max_j \tilde{v}_j \mid \max_j \tilde{v}_j \leq v_j] s_j^*(v).$$

Still satisfies IC, participation and BB.

Has range of $\mathbb{E} \max_j v_j \leq 2\mathbb{E}[\max_{j \neq i} v_j]$.

The existence condition

Define $g(v) = \max_j v_j$ and $C_i := \inf_{v_i \in V_i}\{\mathbb{E}_{-i} g(v) - U_i^0(v_i)\}$.

THEOREM 1. *An efficient mechanism satisfying incentive compatibility, liquidity, participation and budget balance constraints exists* **if and only if**

$$\sum_{i \in N} \min\{C_i, l_i\} \geq (n-1)\mathbb{E}g(v).$$

The modified EEM always works under this condition.

- Means that it has the lowest possible range.

A liquidity-constrained auction

Let $F_i = F$, $V_i = [0, \overline{v}]$, and let $b := (b_1, \dots, b_n) \in \mathbb{R}^n_+$.

PROPOSITION 1. *The liquidity-constrained auction is such that*

- *The good is allocated to the highest bidder;*
- *Agent i pays a price*

$$p_i(b) := \begin{cases} (n-1)b_i & \text{if } \; b_i \geq \max_k b_k \\ -\, b_j & \text{if } \; b_j \geq \max_k b_k, \end{cases}$$

- *Agent i receives a lump-sum transfer $\phi_i(U^0, I)$.*

Always works under the condition of the theorem.

Market size

How does **market size** affect our efficient liquidity-constrained mechanisms?

At first sight, difficult to say: n increases both the LHS and the LHS and RHS of:

$$\sum_{i \in N} \min\{C_i, l_i\} \geq (n-1)\mathbb{E}g(v).$$

PROPOSITION 2. *Assume* $l_i := \tilde{l}$ *for all* i*. An efficient allocation mechanisms exists* **only if**

$$\tilde{l} \geq \frac{n-1}{n}\mathbb{E}g(v).$$

The threshold $\tilde{l}$ *is increasing in* n *and converges to* $\overline{v}$ *when* $n \to \infty$*.*

Market size: Seller-buyers example

Consider an extension of the buyer-seller problem of Myerson and Satterthwaite (1986) to **multiple buyers.**

One seller, $i = 1$, with valuation $v_1 \in [0, c]$ and $u_i^0(v) = v_1$.

- Outside option accounts for ownership of the good.

And $(n - 1)$ buyers with valuation $v_i \in [0, 1]$ and $u_i^0(v) = 0$.

When $n = 2$, the **celebrated result of MS (1986) applies:**

- There exists no efficient allocation mechanism.

Market size: Seller-buyers example

However, when $n > 2$, there exists a threshold $n^*(c)$ **such that efficient trade is possible** if $n \geq n^*(c)$.

Intuition: If there are enough buyers ($\sim$ competition) $\Rightarrow$ Efficient trade is possible.

Objection: The more the buyers, the stronger the liquidity requirements.

For instance, assume uniform distributions and let $c \to 0$ (most favorable case):

# of buyers	1	2	3	4	5	6	7	8	9
$\tilde{l}$	0	.33	.50	.60	.66	.71	.75	.77	.80

Reallocation mechanisms

A particular case of the previous framework is the case of **reallocation** problems.

- Agents initially *owns* a share of the resource.
- We are looking for an efficient reallocation of the resource among them.

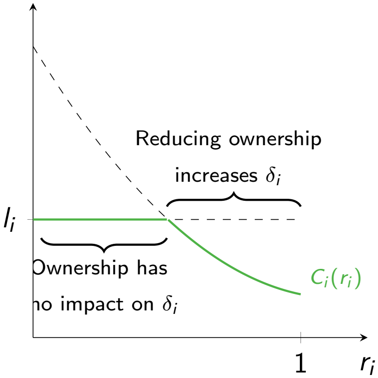
For instance, let $u_i^0(v) = v_i r_i$ be the outside option of agent i.

- $r_i \in [0, 1]$ is agent i's initial ownership share, $\sum_{j \in N} r_j = 1$.
- If agent i refuses to participate: still enjoys their share of the good.

Challenge: The higher the share of an agent, the more difficult it is to make them participate.

Ownership and liquidity

Each agent cannot be charged more than $\delta_i := \min\{C_i(r_i), l_i\}$.



Ownership and liquidity

Lesson: Reducing ownership of agent i increases how much we can charge them...

... up to the point they become liquidity constrained.

PROPOSITION 3. *Let $l_1 \geq \cdots \geq l_n$, efficient reallocation is more likely to be attainable when $r_1 \leq \cdots \leq r_n$.*

Contrasts with the CGK's characterization in which *equal-sharing ownership* is the ownership structure that makes efficient trade feasible.

Concluding remarks

The design of incentive compatibility constraints is crucial to account for liquidity constraints:

- Lower *range* of prices $\Rightarrow$ Redistribution *more effective*.

This work characterizes the minimal liquidity requirements to achieve an efficient allocation.

And shows how market size or initial ownership in reallocation problems affects the *performance* of liquidity-constrained mechanisms.

Thank you!