Competing mechanisms: Competing auctions

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Introduction

Incentive theory has been very successful to deal with asymmetric information issues.

Most prominent example is auction design.

There is a form of competition, but it takes place among the agents (buyers).

The principal (the seller) is often treated as a monopolist.

It is often viewed a normative theory of price/allocations.

Introduction

In other fields, models of competition naturally include several buyers and sellers.

- Industrial organization.

Many sellers compete for buyers by choosing an action in a fixed set of rules.

Bertrand/Cournot competition, two part tariffs, ...

Introduction

What if sellers could not only design their rule of trade and compete for buyers?

- This is the idea of competing mechanisms.

Could we aim at a theory of competition/price that include all those possibilities?

Many challenge on the way...

Mechanisms

First, let us briefly define what we call a mechanism.

A mechanism is a mapping from a set of message to a set of allocations.

Formally, $\gamma : M \to X$.

Roughly speaking, sellers would compete by offering different mechanisms, i.e., rules of trade.

Why does it matter?

First, why would we restrict principals to use a fix set of rules?

- States designing taxation mechanisms.
- Selling a car/house with specific guidelines/requirements.

Could we see emerge real life way to compete from investigating the most general framework?

Also important for possible deviation: Maybe sellers in Bertrand could deviate in other ways that increasing/decreasing their price?

Challenges

This approach raises a number of challenges.

How to describe those environments?

How to deal with this level of generality? Usually the revelation principle helps to get tractable problems.

- Fails or difficult to apply here.

We will investigate those questions and some of the possible answers.

Roadmap

1. Competing auctions: Simplified environment with a large number of buyers and sellers.
2. The revelation principle: Failures and remedies.
3. More on equilibrium allocations and on possible extensions.

Focus on a few major papers who have paved the way for the study of competing mechanisms.

Standard analyses of price formation

As said previously, two main strands of the literature on **price formation**.

Consider a good, n buyers and,

(i) **Many** sellers compete according to a **fixed set of rules.**

v.s.

(ii) A **single** seller **fully designs the set of rules** to allocate the good.

Competing mechanisms

We would like to **bring together** the study of

(i) Competitive markets

(ii) Design of selling mechanisms

That is, allow for **many sellers** to compete for buyers while also allowing them to **design the rules of trade**.

Some of the important questions are the following

1. How do buyers form their participation choices?
2. What is a deviation for a seller?
3. What type of mechanisms sellers offer in equilibrium?
4. What parallel can we draw with more *standard* models?

Technical difficulties

Relaxing the **monopoly position** of a seller fully designing the selling mechanism introduces several **technical difficulties**.

For instance,

- The **incentive compatibility** of seller i's mechanism is tied to mechanisms offered by all other sellers $j \neq i$.
- The choice of seller i's mechanism affects the **surplus available to buyers** who do not participate in seller i's mechanism.

The **complexity** of the problem will unfortunately force us to make some **restricting assumptions**.

Methodology

I will present two *methodologies* to address this problem and its complexity.

1. **McAfee** (1993) and **Peters** (1997)
   a. Large economies
   b. Direct mechanisms

2. **Burguet and Sakovics** (1999)
   a. Finite economies
   b. Restricted class of mechanisms

Methodology: McAfee and Peters

Both rely on the assumption that the **number of sellers and buyers is large**.

- McAfee relies on that to make some **behavioral assumptions** on traders, namely, that they **neglect some of the strategic repercussions of their actions**.
- Peters shows that some of McAfee's behavioral assumptions are actually **implied by** the large economy assumption.

Both restrict to the class of **direct mechanisms**.

- Not without loss of generality (Attar et al., 2018).

Methodology: Burguet and Sakovics

On the contrary, Burguet and Sakovics work with a **finite number of traders**.

They take into account **all the strategic implications** of the agents' actions.

But **restrict themselves** to a *smaller* class of mechanisms.

Still **do not** consider communication without participation in a mechanism.

Common themes

The three most noticeable **common aspects** shared by all three papers are the following.

1. Selling mechanisms are **auctions**.
   - Arise **endogenously** in McAfee and Peters.
   - Posited in Burguet and Sakovics.
2. The **buyers' participation strategies** are key to the analysis.
   - If seller i **deviates**, it naturally affects what buyers can expect from participating in seller i's mechanism.
   - But it also changes what buyers can expect from seller $j \neq i$'s mechanism as both **overall participation decisions and reporting strategies change**.
3. Communication is tied to participation.

Outline

I will present the **setting and main results** of Peters (1997) and Burguet and Sakovics (1999).

- The goal is to provide you with an overview of the modeling assumptions and key results.

McAfee (1993) is similar to Peters (1997).

- More complex model.
- Stronger behavioral assumptions.
- Less *standard* treatment.

Peters (1997): Framework

- J sellers and kJ buyers.
- Each seller has a **single indivisible unit** of a good whose production cost is drawn from G with support $\subseteq [0,1]$.
- Buyers have **unit demand** and **valuations** i.i.d. according to F with support on $[0,1]$.
- All traders are **risk neutral** and utilities write

$$p - y,$$
$$x - p,$$

when p is the trading price, y the seller's valuation and x the buyer's valuation.

Peters: Timing

The timing is as follows.

1. Sellers simultaneously offer **direct mechanisms**.
2. Buyers observe all posted mechanisms and **decide in which to participate** (at most one).
3. Each buyer **reports** a valuation to the mechanism in which they participate.
4. Mechanisms are operated.

Note: No communication if no participation.

Peters: Mechanisms (1)

Sellers offer **direct mechanisms**, i.e, allocation and payment rules as functions of the valuation profile.

The subtlety is that mechanisms **must be defined for any possible number of participating buyers**.

Let $\overline{X} := [0,1] \cup \{x_0^i\}$ be set of buyer i's possible types.

- $[0,1]$ is the valuation space.
- x_0^i corresponds to the type "agent i's does not participate".

By convention, if agent i does not participate in seller j's mechanism the i will "report" x_0^i to this mechanism.

Peters: Mechanisms (2)

Let $\mu^j := \{q^j, p^j\}$ be seller j's mechanism and $\mu := \{\mu^1, \ldots, \mu^J\}$, where

$$q^j : \overline{X}^{kJ} \to [0,1]^{kJ},$$
$$p^j : \overline{X}^{kJ} \to \mathbb{R}^{kJ}.$$

q^j and p^j are the allocation and payment rules, respectively.

Let $q^j(\overline{x}) := (q^{j1}(\overline{x}), \ldots, q^{jkJ}(\overline{x}))$ where $q^{ji}(\overline{x})$ is the probability that seller j allocates the good to buyer i when $\overline{x} \in \overline{X}^{kJ}$ is the vector of reported valuations.

The same applies for p^j.

Peters: Constraints on mechanisms

The following constraints are imposed on any μ^j.

Resource constraints. $\sum_{i=1}^{kJ} q^{ji}(\overline{x}) \leq 1$ for all $\overline{x}$, j.

Anonymity. if $\overline{y}$ is a permutation of $\overline{x}$ then $q^j(\overline{y})$ is a permutation of $q^j(\overline{x})$ in the same way.

"Participation". $\overline{x}^i = x_0^i \Rightarrow q^{ji}(\overline{x}) = 0$.

- Not participating in seller j's mechanism implies zero probability of getting the good.
- This convention ensures that designing μ^j encompasses all possible participation scenarios.

Peters: Reduced-form mechanisms (1)

The usual way to proceed is to defined **reduced-form** mechanisms (interim expected values) of the form $\left\{Q^j(\cdot), P^j(\cdot)\right\}$.

Q^j corresponds to a buyer **expected probability of receiving the good** from seller j only as function of this buyer's valuation. Same for P^j.

The problem here is that **we cannot** simply take expectation of other buyers' valuations according to the distribution of valuations F.

The **actual distribution of valuations** faced by seller j depends both on F and on buyers' participation decisions.

Peters: Reduced-form mechanisms (2)

Let $\pi_j(x, \mu)$ denote the probability that a buyer selects seller j when they have valuation x and posted mechanisms are μ.

Then, the probability that a buyer has a valuation lower than x **or** that they do not participate in seller j's mechanism writes

$$z_j(x, \mu) := 1 - \int_x^1 \pi_j(s, \mu) f(s) ds.$$

This probability defines the **actual distribution of buyers' valuations** that seller j is facing.

Peters: Reduced-form mechanisms (3)

The **reduced-form allocation rule** can therefore be defined as

$$Q^j(x, \mu, \pi) := \int_{[0,1]^{kJ-1}} q^{j1}(x, s_2, \ldots, s_{kJ}) dz_1(s_2, \pi) \ldots dz_1(s_{kJ}, \pi),$$

where we take buyer 1 for convenience (anonymity implies symmetry).

The same applies to P^j.

Peters: Utility

The **expected payoff** of a buyer trading with j writes

$$
\begin{aligned}
v_j(x,\mu,\pi) :&= Q^j(x,\mu,\pi)x - P^j(x,\mu,\pi) \\
&= Q^j(y_j,\mu,\pi)y_j - P^j(y_j,\mu,\pi) + \int_{y_j}^{x} Q^j(s,\mu,\pi)ds.
\end{aligned}
$$

The second line stems from incentive compatibility.

And y_j is the greatest lower bound of the interior of the set $\{x : \pi_j(x,\mu) > 0\}$, that is the **cut-off type** that participates in seller j's mechanism.

Standard results apply to $v_j(\cdot)$: continuous, convex and a.e. differentiable in x with $\partial v_j/\partial x(x,\cdot) = Q^j(x,\cdot)$.

Peters: Incentive consistent participation strategies

Some requirements are needed to specify how buyers match with sellers.

A choice strategy is **incentive consistent** if it satisfies the following.

(i) $\sum_{j=1}^{J} \pi_j(x, \mu) = 1$.

(ii) $\pi_j(x, \mu) = 0 \Rightarrow \exists k \neq j, v_k(x, \cdot) \geq v_j(x, \cdot)$.

(iii) $\pi_j(x, \mu) > 0 \Rightarrow v_j(x, \cdot) \geq v_k(x, \cdot)$ for all k.

If buyer i expects all other buyers to follow π, then has no incentive to choose sellers other than with π.

Peters: Limit game

In order to characterize the equilibrium, we must me able to define what is a **deviation** from a seller.

The main difficulty is that when seller j deviates it

(i) Changes the distribution of buyers going to j.
(ii) May also affect the distribution of buyers going to non-deviating sellers.

Computing **deviation payoffs** becomes extremely difficult.

By assuming that $J \to \infty$, Peters is able to escape the problem and characterize an **upper bound on deviation payoffs**.

Peters: Main result

The main result is that when $J \to \infty$, there exists an equilibrium in which each seller of type w holds a **second-price auction with reserve price** w.

Same result as in McAfee but **without making behavioral assumptions on traders** neglecting some strategic effects.

- The large number of traders is enough to prove the result.

More generally, McAfee shows that sellers hold *auctions*, not necessarily second-price auctions.

- Where an auction is simply defined as **a mechanism in which the highest valuation agent must receive the good.**

Peters: An "intuition" of the proof

Peters establishes a mapping from **cutoff valuations** into sellers' payoffs and reserve prices that they support.

Using the *large economy* assumption, it is possible to construct a finite approximation of the **limit distribution of cutoff valuations**.

Then shows that the **distribution of reserve prices converges to a fixed distribution** even if one of a single seller **deviates** and chooses another mechanism.

- Non-deviating sellers' payoffs converge to a function independent of the deviator's mechanism.

The payoff of a deviator is **bounded above** by the payoff of holding an auction with reserve price w for any given fixed distribution of reserve prices.

Peters: Some interpretations

In the **monopoly auction setting**, the seller designs an auction that **extracts rents** from the buyers.

- The reserve price **depends on the seller's beliefs** about buyers' valuations.

In the competitive case, reserve prices are **independent of beliefs**: $r_j = w_j$.

Mechanisms are not as fine-tuned to beliefs under competitive pressure as they are in a monopoly.

Peters: Some interpretations

Because of the presence of other sellers, a seller is **limited in their ability to raise their reservation price**.

- Excluding low valuation buyer is still beneficial;
- But is more limited than in the monopoly case because buyers can join other mechanisms.

It may seem that sellers makes zero profit as they set their reserve price to their cost.

- But they still enjoy positive profits when buyers with higher valuations than the reserve price participate.

Burguet and Sakovics: Setting

- Two sellers, 1 and 2, each has one unit of an homogeneous good.
- N ex ante symmetric buyers with i.d.d. valuations $v_i \sim F(v_i)$ with support on $[0, 1]$.
- Sellers are restricted to **second-price auctions** $\Rightarrow$ only strategic variable is their **reserve price**.
- Buyers observe the posted reserve prices and decide in which auction to participate.

Finite number of traders but restrictive assumptions on trading rules (partially relaxed later on).

BS: Buyers' equilibrium behavior

Wlog assume $r_1 \leq r_2$.

Notice that, conditional on participating to auction $i = 1, 2$, **reporting their valuation is a dominant strategy** for each buyer.

- Second-price auction.

BS: Buyers' equilibrium behavior

To get some intuition, consider a buyer with valuation v.

- $v < r_1$: Optimal not to participate in any auction.
- $r_1 \leq v \leq r_2$: Optimal to participate in 1.
- $v = r_2 + \epsilon$: Optimal to participate in 1.
    - Almost null profits in 2.
    - Positive profits in 1.
- If $r_2 << 1$: Some buyers may participate in 2.
    - If no one participates in 2,
    - and some buyers have v close to 1,
    - participating in 2 yields $v - r_2$,
    - which might be better than participating in 1.

BS: Buyers' equilibrium behavior

The equilibrium behavior is the following.

For given $r_1 \leq r_2$ and some $w \in [r_2, 1]$,

- $v < r_1$: No auction.
- $r_1 \leq v < w$: Auction 1.
- $v \geq w$: Auction $i = 1, 2$ with prob $1/2$.

w is uniquely defined, decreasing in r_1 and increasing in r_2.

- For each seller, increasing their reserve price reduces their "demand".

BS: Sellers' equilibrium

Given the buyers' equilibrium behavior, let us investigate the sellers' choice of reserve prices.

First result. There exists no symmetric pure-strategy equilibrium for the sellers.

When $r_1 = r_2 > 0$, then each seller has an incentive to **slightly lower** their reserve price.

When $r_1 = r_2 = 0$, then each seller has an incentive to **increase** their reserve price.

BS: Sellers' equilibrium

Second result. There exists a (mixed-strategy) equilibrium for the sellers' game. And the probability that any $r_i = 0$ is null.

This means that **competing in "prices"** (like in Bertrand) will never lead to zero reserve prices ex post.

Different from Peters (1997) in which sellers set their reserve price equal to their cost (which is normalized to 0 here).

- Peters' result is a consequence of the large economy assumption.

BS: Larger class of mechanisms

Results are partially generalized to the class of **quasi-efficient mechanisms**.

A mechanism is quasi-efficient if it allocates the good efficiently **conditional upon the buyers' attendance decisions.**

In other words, buyers must have a **strictly monotone bidding strategy** in the mechanism they participate in.

- First-price auctions are a candidate.
- And of course second-price auctions.

BS: Larger class of mechanisms

Buyers' equilibrium behavior is similar to the restricted mechanism case.

There exists two cut off points $0 \leq w_1 \leq w_2 \leq 1$ such that

- $v < w_1$: Do not participate at all.
- $w_1 \leq v < w_2$: All select the same seller.
- $v \geq w_2$: Visit each seller with equal probability.

BS: Larger class of mechanisms

Extension of the previous result. If sellers can choose among quasi-efficient mechanisms, the case $w_1 = w_2 = 0$ cannot constitute an equilibrium for the sellers.

In other words, some buyers will be excluded at equilibrium.

Those results indicates that there is a **divergence** between a model of competing auction and the standard oligopoly models of price competition.

Virag (2010)

Virag (2010) fills the gap between Peters (1997) and Burguet and Sakovics (1999).

In short, Virag extend BS's result to any **finite** number of sellers.

And shows that as the number of sellers become large, **sellers posted reserve prices converge to 0** (in distribution).